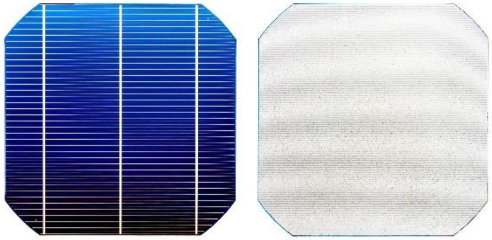


Wykład 5
23.04.2024 r.
Numeryczne rozwiązanie równania Poissona dla złącza Schottky'ego

Katarzyna Gwóźdź

Równanie Poissona

$$\frac{\mathrm{d}^2V(x)}{\mathrm{d}x^2} = -\frac{\rho(x)}{\varepsilon_0\varepsilon_r}$$

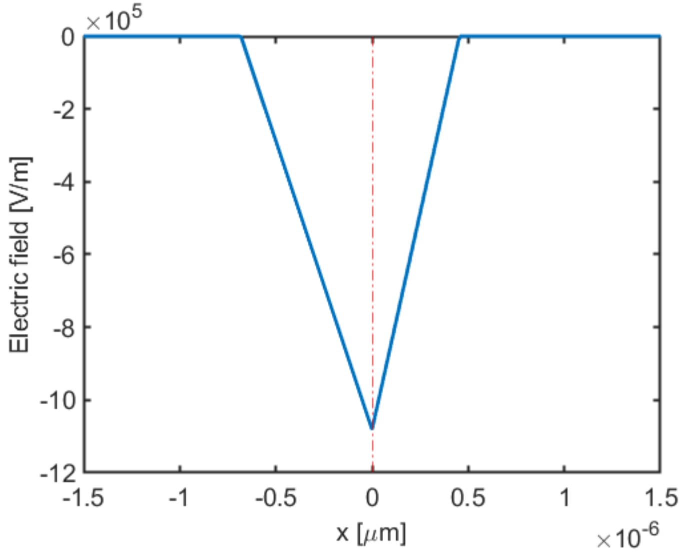
$$\rho(x) = qN_d\left(1 - \exp\left(\frac{qV(x)}{kT}\right)\right)$$

Przybliżenie obszaru całkowicie zubożonego:

$$\rho(x) = \begin{cases} -qN_a, & obszar\ p \\ qN_d, & obszar\ n \end{cases}$$

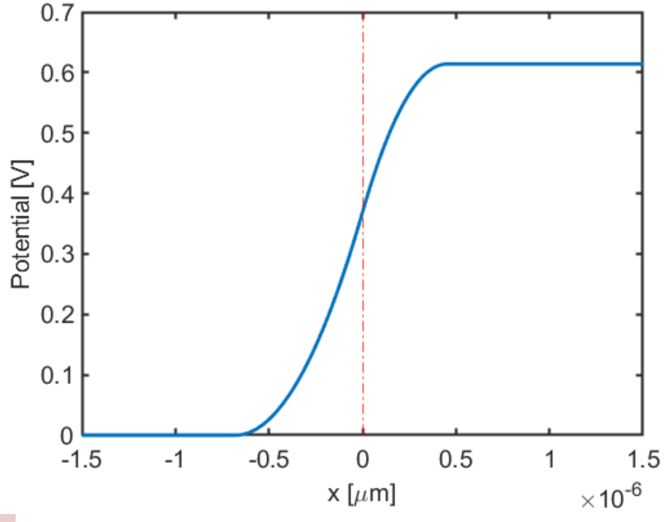
Pole elektryczne

$$E(x) = \begin{cases} -\dfrac{qN_a}{\varepsilon}(x_p + x), & obszar\ p \\ -\dfrac{qN_d}{\varepsilon}(x_n - x), & obszar\ n \end{cases}$$



Potencjał

$$V(x) = \begin{cases} \frac{qN_a}{\varepsilon}\left(x_p + \frac{x}{2}\right)x + \frac{qN_a}{2\varepsilon}x_p^2, & obszar\ p \\ \frac{qN_d}{\varepsilon}\left(x_n - \frac{x}{2}\right)x + \frac{qN_a}{2\varepsilon}x_p^2, & obszar\ n \end{cases}$$



Numeryczne rozwiązanie równania Poissona

$$\rho(x) = qN_d\left(1 - \exp\left(\frac{eV(x)}{kT}\right)\right)$$

$$\frac{\mathrm{d}^2V(x)}{\mathrm{d}x^2} = -\frac{qN_d\left(1 - \exp\left(\frac{eV(x)}{kT}\right)\right)}{\varepsilon_0\varepsilon_r}$$

$$\frac{dE(x)}{\mathrm{d}x} = \frac{\rho(x)}{\varepsilon_0\varepsilon_r}$$

$$\frac{dV(x)}{\mathrm{d}x} = -E(x)$$

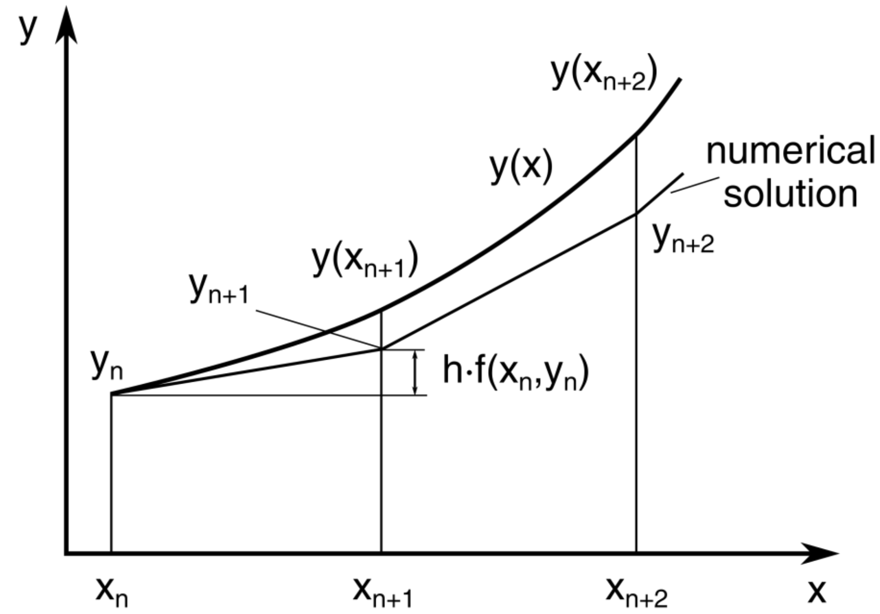
Metoda Eulera

$$\frac{dy}{dx} = f(x, y(x)) \qquad y(x_0) = y_0$$

$$\frac{dy}{dx} = f(x_i, y_\mathrm{i})$$

$$y_{i+1} = y_i + f(x_i, y_i) * h$$

$$h = x_{i+1} - x_i$$

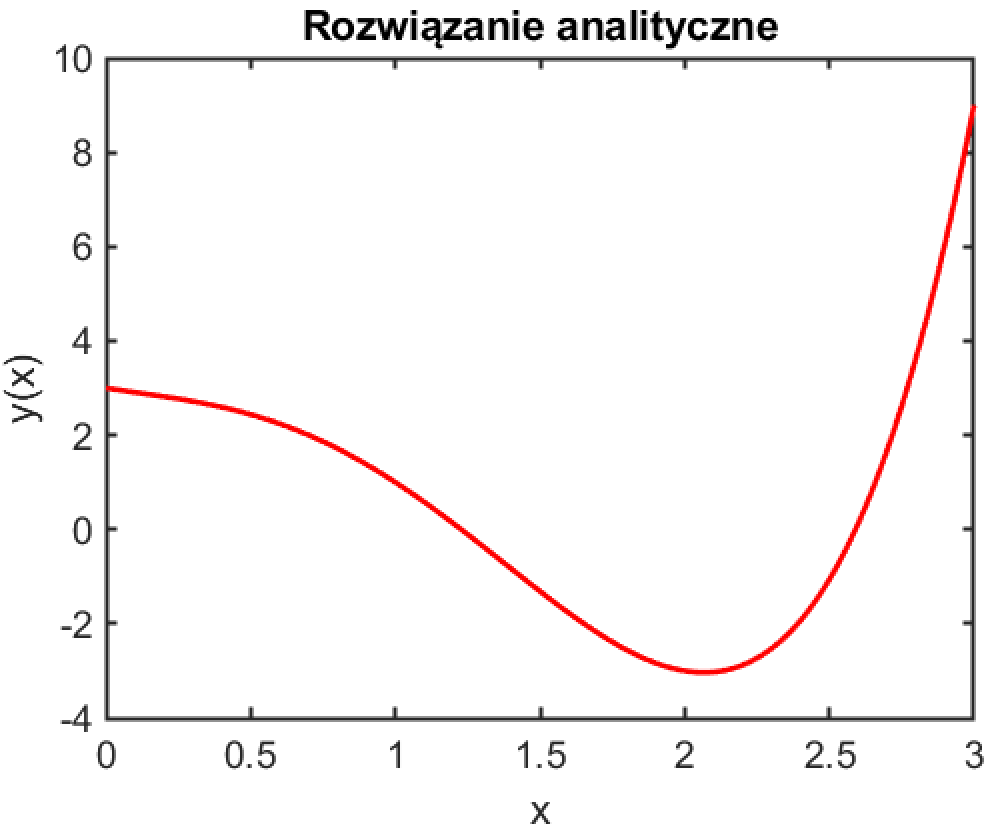


Przykład

$$\frac{dy}{dx} = 4x^3 - 9x^2 + 2x - 1 \qquad y(0) = 3$$

$$y = x^4 - 3x^3 + x^2 - x + C \qquad C = 3$$

$$y = x^4 - 3x^3 + x^2 - x + 3$$



Przykład

$$\frac{dy}{dx} = 4x^3 - 9x^2 + 2x - 1 \qquad y(0) = 3 \qquad x_1 = 0 \qquad y_1 = 3$$

$$y_{i+1} = y_i + f(x_i, y_i) * (x_{i+1} - x_i)$$

$dla\ x_2 = 1$

$y_2 = y_1 + f(x_1, y_1) * (x_2 - x_1) = 2$

$y_1 = 3$

$f(x_1, y_1) = 4 * 0^3 - 9 * 0^2 + 2 * 0 - 1 = -1$

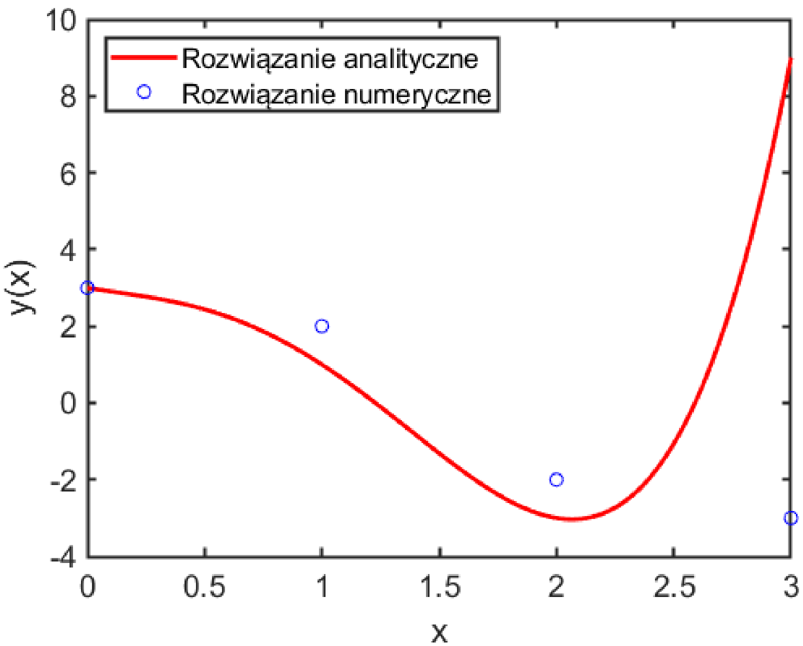
$h = (x_2 - x_1) = 1 - 0 = 1$

$dla\ x_3 = 2$

$y_3 = -2$

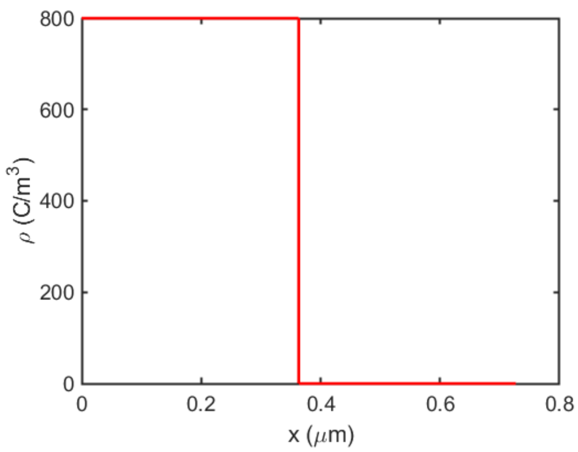
$dla\ x_4 = 3$

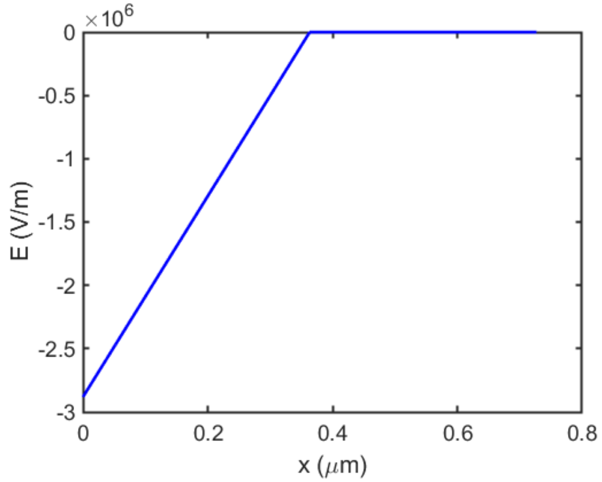
$y_4 = -3$

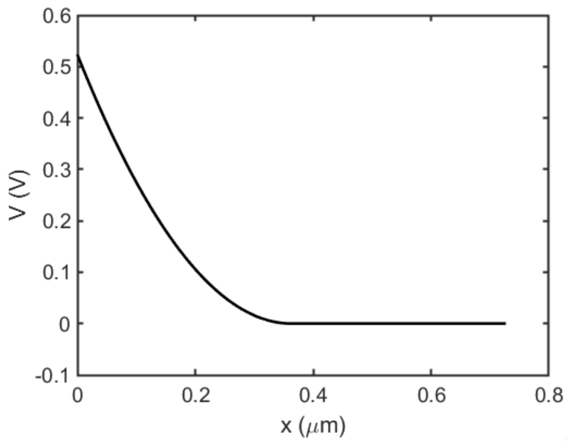


Warunki początkowe

$$L_D = \sqrt{\frac{\varepsilon kT}{q^2 N_d}}$$







$$dla\ x \gg x_d$$

$$V \approx 0$$

$$\rho = 0$$

$$E = 0$$

$$V_{end} = -\frac{kT}{q} exp\left(\frac{-x_d}{L_D}\right)$$

Odwrotna metoda Eulera

$$y_i = y_{i+1} + f(x_{i+1}, y_{i+1}) * h$$

$$h = x_i - x_{i+1} \qquad h < 0$$

Wskazówki

$$\rho(i) = eN_d\left(1 - \exp\left(\frac{eV(i+1)}{kT}\right)\right)$$

$$E(i) = E(i+1) + h * \frac{\rho(i)}{\varepsilon_0\varepsilon_r}$$

$$h < 0$$

$$V(i) = V(i+1) - h * E(i)$$

Vbi -> patrz zadanie 1

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon}{q}V_{bi}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$N_a \to \infty$$